TYBSC Sem-VI 12/4/2023

N.B.(i) All questions are compulsory.
(ii) Figures to the right indicate full marks.

Time : 3 Hours

Marks : 100

Q.1 (a) Suppose (X, Y) follows Bivariate Normal Distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, derive marginal distribution of X and marginal distribution of Y. **(10)**

(b) Suppose (X, Y) follows Bivariate Normal Distribution with parameters $(0, 0, 1, 1, \rho)$. Obtain its joint MGF and hence marginal distributions of X and Y. **(10)**

OR

(p) Define fisher's Z transformation and explain its use in testing **(10)**
   i. $H_0: \rho = \rho_0$ against $H_1: \rho > \rho_0$
   ii. $H_0: \rho = \rho_0$ against $H_1: \rho < \rho_0$
   iii. $H_0: \rho = \rho_0$ against $H_1: \rho \neq \rho_0$ for large samples, in case of bivariate normal population.
   Also obtain 95% confidence interval for ρ.

(q) Let X, Y and Z be i.i.d. Normal(0,1). **(10)**
   i.   Show that Joint distribution of $U = X + aZ$ and $V = Y + aZ$ is Bivariate Normal.
   ii. In above example, find 'a' such that correlation coefficient between U and V is $\frac{1}{2}$

Q.2 (a) Let X be a discrete r.v. such that $P(X=n) = P_n$ $n = 0,1,2\ldots\ldots..$ **(10)**
   Obtain generating functions of the following sequences.
   i. $q_n = p(X \geq n)$
   ii. $q_n = p(X < n)$

(b) i. Define Probability Generating Function (PGF) of discrete random variable assuming integral values **(02)**
   ii. Obtain PGF of Negative Binomial distribution and hence derive its mean and variance. **(08)**

OR

(p) i. A uniform die is thrown. Let X denote number on uppermost face of a die. Find PGF of X. **(04)**
   ii. Bernoulli trial is repeated till success is occurred K times. Let X be the number of tra=ilas required to get k^{th} success. Obtain PGF of X and mean. **(06)**

(q) i. Let $x_1, x_2, \ldots, x_n$ be n independent random variables with PGFs $P_1(S), P_2(S), \ldots, P_n(S)$ respectively. Obtain the PGF of $\sum x_i$. **(10)**
   ii. Let U(S) and V(S) be the generating functions of two independent random variables, respectively. Obtain the generating functions of (a) X + Y and (b) X – Y

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Q.3 (a) In the usual notations, for Poisson birth process, list the postulates, state the (10) expression $P_n(t)$, the probability of 'n' numbers in the system at time 't' and find its mean and variance.

(b) Stating clearly postulates for the pure death process, with initially 'i' members (10) in the system at time t=0. Derive the difference differential equation for $\mu_n = n\mu$ and find the expression for the probability of 'n' numbers in the system at time 't'.

OR

(p) For linear growth model having birth rate $(n\lambda)$ and death rate $(n\mu)$ and initially (10) at time $t = 0$, there are 'a' member in the system then
    i.   List the Postulates.
    ii.   Derive difference differential equation.

(q) In usual notation for Poisson death process, list the postulates, state the expression (10) for $P_n(t)$, probability that 'n' members in the system at time 't'. Find its mean and variance.

Q.4 (a) Define the following terms; (10)
    (i)   Input Process
    (ii)   Server and customer
    (iii)   Reneging
    (iv)   Jockeying
    (v)   Balking

(b) Show that for a single service station with Poisson arrival and exponential service (10) time the probability that exactly 'n' calling units are in the queue system is,
$$P_n = (1 - \rho)\rho^n \quad ; n \geq 0$$
Where, ρ is the traffic intensity.

OR

(p) For $\{(M/M/1) : (N/FIFO)\}$ queueing model, derive the expression for P_n and find (10) $E(n)$.

(q) Discuss the classification of queuing model, and operating characteristics of (10) queueing system.

Q.5   Attempt Any Two sub questions

(a) Show that (X, Y) follows Bivariate Normal Distribution with parameters **(10)** $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ if and only if every linear combination of X and Y viz. $ax+by$, $a \neq 0$, $b \neq 0$, is a Normal Variate. Where a and b are constants.

(b) Let X be a r.v. assuming values 0, 1, 2, ... with probabilities $p_0, p_1, p_2, \ldots\ldots$ Let **(10)** $q_j = P(X > j)$ and $p_j = P(X = j)$, $j = 1, 1, 2, \ldots.$ Assuming actual notations, show that $Q(S) = \frac{1-P(S)}{1-S}$ where $Q(S) = \sum_{j=0}^{\infty} q_j S^j$ and $P(S) = \sum_{j=0}^{\infty} p_j S^j$. Also obtain mean and variance of X in terms of Q.

(c) For pure birth process, if the growth rate λ_n is directly proportional to the number **(10)** of individuals $(n \geq 1)$ say $\lambda_n \propto n$, assuming no death or removal and initially at time t=0, there are 'i' members in the system, then

    i. Obtain difference differential equation.

    ii. Find $P_n(t)$

(d) What are the service disciplines? Describe some forms of common service **(10)** Disciplines and illustrate with examples.