

**UNIVERSITY OF MUMBAI**  
**SEMESTER VI REGULAR/ATKT PRACTICAL EXAMINATION, Apr-2023**

Subject: STATISTICS

Subject Code : USSTP07 (Paper I & Paper II)

Date : 25/04/2023

Time: 9.00 a.m. to 12.00 noon

Max. Marks : 80

- N. B.1. Attempt any ONE question from Q.1 & Q.2 of PART A & any ONE question from Q.3 & Q.4 of PART B  
 2. Use of Statistical Tables & Scientific Calculator is allowed.  
 3. Each Question is of 40 marks and all Sub questions in any question carry equal marks.  
 4. Draw neat and labelled diagram wherever necessary.

**PART A**

**Solve the following**

(40)

Let X and Y have a Bivariate Normal Distribution with parameters  $\mu_1 = 3, \mu_2 = 0,$

$$\sigma_1^2 = 16, \sigma_2^2 = 25, \rho = 0.6$$

Determine the following probabilities.

- i)  $P(2 < Y < 7)$
- ii)  $P(2 < Y < 7 / X = 7)$

Data was collected on time spent in Supermarket in minutes (X) and amount spent in hundreds (Y) for 11 people.

|   |    |    |    |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|----|----|----|
| X | 35 | 40 | 22 | 50 | 37 | 42 | 10 | 15 | 20 | 43 | 18 |
| Y | 12 | 13 | 10 | 22 | 30 | 15 | 5  | 10 | 15 | 20 | 8  |

Obtain coefficient of correlation. Test its significance

An office opens at 11 a.m. daily. There are 2 particular employees who come before 11 a.m. and other employees arrive at the rate of 2 employees per minute after 11 a.m. Obtain the probability that there are

- (i) Less than 3 employees at 11.01 a.m.
- (ii) 10 employees at 11.05 a.m. Also obtain expected number of employees at 11.02 a.m.

The inter arrival time at a single server tool crib is exponential with an average time of 12 minutes. The length of service time is assumed to be exponentially distributed with mean 9 minutes. Obtain

- a. Probability that an arriving worker has to wait
- b. Average queue length
- c. Average length of non-empty queue.

OR

**Solve the following**

(40)

Let X and Y have a bivariate normal distribution given by

$$f(x, y) = \frac{1}{6\pi\sqrt{7}} \exp \left[ -\frac{8}{7} \left( \frac{x^2}{16} - \frac{3xy}{32} + \frac{xy}{8} + \frac{y^2}{9} - \frac{4y}{3} + \frac{71}{16} \right) \right]$$

Obtain

- (i) Parameters of the distribution;
- (ii) Conditional Distribution of Y given  $X=x$ ;
- (iii)  $P(4 < Y < 6 / X=6)$

- (B) A test in Mathematics was given to 10 students who were about to begin the course in Statistics. The scores in the tests (X) were examined in relation to scores in final examination in Statistics (Y). Following results were obtained.

$$\sum X=71, \sum Y=70, \sum X^2=555, \sum Y^2=526, \sum XY=527$$

Obtain the correlation coefficient between X and Y and test whether value of corresponding population correlation coefficient is 0.85

- (C)  $X_1, X_2, X_3, \dots, X_9$  are mutually independent random variables having following probability mass function

$$P(x) = \frac{1}{3} \left(\frac{2}{3}\right)^{1-x}; x=0,1$$

$$= 0, \text{ otherwise}$$

Obtain

- i) Probability generating function of  $X_i$

Probability generating function of  $Y = \sum_{i=1}^9 X_i$

- (D) A petrol pumping station serves only cars. It has three pumps and a maximum of seven cars in all are permitted in the station. During peak hours arrivals follow Poisson distribution with mean of one car per minute. The service time is exponential with mean of 6 minutes. Obtain average number of cars in the system and average time spent by a car in the system during peak hours.

### ✓ PART B

Solve the following

(40)

Q.3.

(A)

An urn contains 20 marbles of which an unknown number  $\theta$  are white in colour. To test the hypothesis:  $\theta$  is less than or equal to 3 against an alternative hypothesis:  $\theta$  is greater than 3, two marbles are chosen without replacement and the null hypothesis is rejected if one of them is white. Find  $\alpha$  and power of the test at  $\theta=5$  and 10. Also find level of significance

(B)

Let  $X \sim \text{Geometric distribution } (\theta)$

$$P(x) = \theta(1-\theta)^{x-1} \quad x=1,2,3,4,\dots$$

Find Best critical region and Most powerful test of size 0.01 to test  $H_0: \theta = 0.4$ .

$H_1: \theta < 0.4$  at 1% level of significance on the basis of a single observation. Is this test uniformly most powerful?

(C)

Let X be a random variable having probability mass function (p.d.f.)

$$f(x, p) = p^x (1-p)^{1-x}; x=0,1$$

$$= 0; \text{ otherwise}$$

Derive SPRT of strength  $\alpha = 0.3$  and  $\beta = 0.5$  for testing  $H_0: p = 2/5$  against  $H_1: p = 3/5$ . What will be the conclusion if the first sample observation has value 0?

(D)

Changes in rat cerebral and extra cerebral serotonin after administration of LSD 25 and 10 UML were checked. Measurements on 11 controls were also taken. The results have been shown in the table given below. Do these data groups provide sufficient evidence to indicate a difference between the three groups? Use Kruskal Wallis test at 5% level of significance.

| Control | LSD(0.5mg/kg) | UML(0.5mg/kg) | Control | LSD(0.5mg/kg) | UML(0.5mg/kg) |
|---------|---------------|---------------|---------|---------------|---------------|
| 340     | 294           | 263           | 402     | 371           | 371           |
| 340     | 325           | 309           | 402     | 385           | 402           |
| 356     | 325           | 340           | 417     | 402           | 417           |
| 386     | 340           | 356           | 433     |               |               |
| 386     | 356           | 371           | 495     |               |               |



Q.4. Solve the following

(40)

- (A) The length of a tube has exponential p.d.f. with mean  $\theta$ . In order to test  $H_0: \theta = 8$  against  $H_1: \theta \neq 8$ , consider the critical region of the form  $C = \{X: X < c_1 \text{ or } X > c_2\}$ , where  $c_1 < c_2$ . Find  $c_1$  and  $c_2$  such that each tail has same probability and the size of the test is 0.05. What is the power function of the test? Find power at  $\theta = 5, 9, 13, 15$ .
- (B) I) Suppose that an employee's commute time to work is normally distributed with a mean of 40 min and variance of 100 min. In order to reduce the variability in commuting time so that the employee doesn't reach late for work, he tries an alternative route. 20 such days gave a sample variance of 64. Does the data show evidence at 5% level of significance that the commute time is less variable?
- II) At a cereal filling plant, quality control engineers do not want variance of weights of 750 gm cereal boxes to exceed 100 gms square. A sample of 7 boxes of a type of cereal of printed weight 750 gms was chosen and the actual weights were as follows:  
775, 780, 781, 795, 803, 810, 823.  
Does this sample provide strong evidence at 1% level of significance that the true variance of weights exceeds 100 grams?
- (C) A random variable  $X$  follows a binomial distribution ( $n = 10, p$ ). Derive SPRT of strength  $10 \alpha = 0.1$  and  $\beta = 0.3$  for testing  $H_0: p = 1/2$  against  $H_1: p = 1/3$ .  
What will be your decision if the sample observations are 3, 0, 1 and 6? Also represent the test graphically.
- (D) A research team wants to know whether a new teaching method increases literacy of children. 20 students were selected and their measure of literacy on a scale from 0 to 10 (10- high, 0-low) was noted.  
Then the new method was applied and measure of literacy was again noted  
Use Wilcoxon matched pair sign rank test. To test whether the new method is better than earlier one.

|   | 1 | 2 | 3  | 4 | 5 | 6 | 7 | 8 | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|----|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| X | 1 | 5 | 4  | 5 | 5 | 0 | 5 | 5 | 0  | 0  | 5  | 0  | 6  | 6  | 0  | 0  | 0  | 1  | 2  | 2  |
| Y | 7 | 7 | 10 | 9 | 8 | 3 | 2 | 2 | 10 | 10 | 6  | 10 | 10 | 6  | 10 | 5  | 10 | 8  | 9  | 9  |

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