P-72-Exam. 56-110-4pp-4-2023.

UNIVERSITY OF MUMBAI
SEMESTER VI REGULAR/ATKT PRACTICAL EXAMINATION, Apr-2023
Subject: STATISTICS
Subject Code : USSTP08 (Paper III & Paper IV)
Date : 26/04/2023

Time: 9.00 a.m. to 12.00 noon **Max. Marks : 80**

N. B. 1. Attempt any **ONE** question from **Q.1 & Q.2** of **PART A** & any **ONE** question from **Q.3 & Q.4** of **PART B**
2. Use of Statistical Tables & Scientific Calculator is allowed.
3. Each Question is of **40 marks** and all Sub questions in any question carry equal marks.
4. Draw neat and labelled diagram wherever necessary.

PART A (40)

Q.1 **Solve the following:**

(A) A manufacturer can produce four products using three resources namely Labour, Raw material and Machine time. The unit contributions and the technological constraints are given below in the form of linear programming problem :

Maximize $3x_1 - x_2 + 4x_3 + 7x_4$

Subject to $x_1 + x_2 + x_3 + x_4 \leq 9$ ..... (Labour)

$5x_1 + 3x_2 + 2x_3 + x_4 \leq 60$ .... (Raw material)

$x_1 + 3x_2 + 5x_3 + 8x_4 \leq 50$ .... (Machine time)

$x_i \geq 0 \quad i = 1, 2, 3, 4.$

Optimum simplex table :

		C_j	3	-1	4	7	0	0	0	
C_B	X_B	Basis	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_0
3	X_1	P_1	1	$\frac{5}{7}$	$\frac{3}{7}$	0	$\frac{8}{7}$	0	$\frac{-1}{7}$	$\frac{22}{7}$
0	X_6	P_6	0	$\frac{-6}{7}$	$\frac{-5}{7}$	0	$\frac{-39}{7}$	1	$\frac{4}{7}$	$\frac{269}{7}$
7	X_4	P_4	0	$\frac{-2}{7}$	$\frac{4}{7}$	1	$\frac{-1}{7}$	0	$\frac{4}{28}$	$\frac{41}{7}$
	$Z_j - C_j$		0	$\frac{8}{7}$	$\frac{9}{7}$	0	$\frac{17}{7}$	0	$\frac{4}{7}$	

Using sensitivity analysis, answer the following :

e) For each of the variables X_1 and X_3, find an interval for its objective function coefficient such that the current basic feasible solution remains optimal.

f) If the availability of Machine time resource is decreased from 50 to 8 units, what would happen to the solution ? (check the feasibility of the new solution). Find the new optimal basic feasible solution.

g) Suppose the manufacturer proposes a new product (say X_8 units) whose production coefficients in the constraints for the resources labour, raw material and machine time are 7/4, 3 and 7 respectively and objective function coefficient is 5. Is it worth introducing the product?

h) Determine the ranges over which the resource constraints for the resources labour and raw material can vary without affecting the feasibility of the solution.

(B) A Co. is producing at the rate of 200 units per month and the consumption rate is 100 units per month. The set up cost is Rs.2500/- and the holding cost is 20 Rs per unit per month. i)What is the economic lot size? ii) Find the length of cycle and the cost of this inventory system per cycle. Further suppose the shortages are allowed and the shortage cost is Rs.100/- per unit per month.

What will be the economic lot size and length of production cycle?

(C) A truck is priced at Rs.60,000 and running costs are estimated at Rs.6,000 for each of the first four years, increasing by Rs.2,000 per year in the fifth and the subsequent years. If money is worth 10% per year, when should the truck be replaced? (No scrap value)

(D) A company manufactures around 150 mopeds. The daily production varies from 146 to 154 depending upon the availability of raw materials and other working conditions.

Production per Day	Probability
146	0.04
147	0.09
148	0.12
149	0.14
150	0.11
151	0.10
152	0.20
153	0.12
154	0.08

The finished mopeds are transported in a specially arranged lorry accommodating only 150 mopeds. Using random numbers 80, 81, 76, 75, 64, 43, 18, 26, 10, 12, simulate the process to find out :

the average number of mopeds waiting in the factory,

the average number of empty spaces on the lorry.

OR

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Q.2 Solve the following:

(A) Solve the following Linear Programming Problem (LPP) using Dual Simplex method:

Minimize $Z = 2x_1 + 3x_2 + x_3 + 2x_4$

Subject to $2x_1 + 4x_2 + x_3 + 5x_4 \geq 12$

$x_1 + 3x_2 + x_3 + 6x_4 \geq 4$

$2x_1 + 5x_2 + 3x_3 + 7x_4 \geq 21$

$x_1, x_2, x_3, x_4 \geq 0$

(B) (a) A firm is to order a new lathe. Its power unit is an expensive part and can be ordered only with the lathe. Each of these units is uniquely built for a particular lathe and cannot be used on any other. The firm wants to know how many spare units should be incorporated in the order for each lathe. Cost of the unit when ordered with the lathe is Rs.700. If a spare unit is needed (because of its failure during service) and is not available, the whole lathe becomes useless. The cost of the unit made to order and its downtime cost of lathe is Rs.9300. The analysis of 100 similar units on similar lathes yields the information given in the following table:

No. of spare units reqd :	0	1	2	3	4	5	6	7 or more
Estimated probability:	0.87	0.05	0.03	0.02	0.01	0.01	0.01	0.00

How many spare units should be ordered along with new lathe?

(b) If in the above problem the shortage cost of the part is unknown and the firm wants to maintain stock level of 4 parts find the permissible range of the shortage cost.

(C) Machine A costs Rs.80,000. Annual operating costs are Rs.2,000 for the first year and increasing by Rs.15,000 every year. Determine the least age at which to replace the machine. (Assume that the resale value of the machine is zero.)
(b) Another machine B costs Rs.1,00,000. Annual operating cost for the first year is Rs.4,000 and then increase by Rs.7,000 every year. The firm has a machine of type A which is one year old. Should the firm replace it with B and if so when?

(D) A system has 2 components connected in parallel. The reliabilities of the components are 0.92 and 0.88 respectively at t= 2000 hrs. Determine the system reliability at time 2000 hours of operation. What is the MTTF of the system? (Assume exponential life time distribution.)

Q.3 Solve the following: **PART B**

(A) Complete the following table: (40)

Age(x)	l_x	d_x	$1000q_x$	L_x	T_x	e^0_x
25	90132	-	2.86	-	-	-
26	-	-	2.95	-	-	-
27	-	-	3.01	-	-	-
28	-	-	3.12	-	-	-
29	-	-	3.26	-	-	-
30	-	-	3.49	-	3482464	-

(B) A has a right to receive an amount Rs. 10,000 at the end of 12 years from now. This right has been sold to B for a present value calculated at the rate of 8% p.a. The money thus received was invested by A in deposit account at 9% p.a. payable half yearly. After 8 years the account had to be closed and A then invested the amount available at 6% p.a. in another bank. How much has A gained or lost in this transaction at the end of 12 years?

(C) Find the number of persons living at age 30 to 33 years, if at 6% rate of interest
$a_{30} = 15.045$ $a_{31} = 14.960$ $a_{32} = 14.89$ $a_{33} = 14.80$ $a_{34} = 14.70$ $l_{34} = 956695$.

(D) Find the level annual premium halving after 10 years for a whole life assurance of Rs.5000 for a life aged 28 on the basis of LIC (1970-73) Ultimate mortality table.

OR

Q.4 Solve the following: (40)

(A) If $l_{50} = 937753$, $l_{60} = 859916$ and $l_{70} = 682859$, fit Gompertz law of mortality.

(B) A borrower is repaying one debt of Rs. 4,00,000 by 30 equal half yearly installments of principal and interest calculated at 9% p.a. convertible half yearly of which the 12^{th} has just being paid and another debt of Rs. 1,50,000 by 20 half yearly installments of principal and interest calculated at 8% p.a. convertible half yearly of which the 8^{th} has just been paid. If the remaining installments of the two debts are to be replaced by an annuity certain 20 half yearly payments calculated at interest rate of 10% p.a. payable half yearly, find the installment of the annuity

(C) Find the value of pure endowment benefit of Rs.50,000/- for a person aged 40 years payable on his surviving to age 50 at 6% p.a. given that $a_{40:10\urcorner} = 8.159$ and $a_{40:9\urcorner} = 7.529$ at 8% p.a. rate of interest

(D) Consider following data

x	25	26	27	28	29	30
l_x	97380	97088	96794	96496	96194	95887
d_x	292	294	298	302	307	313

Using the above data find the value of assurance of Rs.25,000/- for a person aged 25 under which the sum assured is payable in case of death between ages 28 and 30 or on survival to age 30 years, rate of interest being 6% p.a

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