

TYB.SC VI 18/4/2023

(Time: 3 hours)

[Total marks 100]

- Note: 1. All questions are compulsory.
 2. Figure to the right indicates full marks.
 3. Use of calculator is permitted.

- Q.1. (a) (i) Define the term force of mortality (μ_x). By making suitable assumptions prove that $\mu_x = \frac{(l_{x-1} - l_{x+1})}{2 l_x}$ (06)
- (ii) Define central death rate (m_x). Show that $p_x = \frac{2 - m_x}{2 + m_x}$ (04)
- (b) (i) Define the term curate expectation of life (e_x) and complete expectation of life (e_x^0). Further in usual notations, show that – (08)
- (1) $e_x = \frac{\sum_{i=1}^{\infty} l_{x+i}}{l_x}$ (2) $e_x^0 = \frac{1}{2} + e_x$ (02)
- (ii) If $l_x = 100 \sqrt{100 - x}$ then find probability of person with age 10 years will survive 5 more years. OR
- (p) (i) State and prove Makeham's first law of mortality. (06)
- (ii) Explain the following terms- (04)
- (1) Select mortality.
 (2) Aggregate mortality.
- (q) Prove that- The average age at death of those who die in the age group x to $x + n$ is given by, $x + \frac{T_x - T_{x+n} - n l_{x+n}}{l_x - l_{x+n}}$ (10)
- Q.2. (a) Define nominal rate of interest ($i^{(m)}$) and effective rate of interest (i). Further in usual notations, show that $i^{(m)} = m [\sqrt[m]{1+i} - 1]$ (10)
- (b) Obtain an expression for present value and accumulated value of immediate annuity of Rs. 1 p.a. when successive payments are in Arithmetic progression. (Assume rate of interest is i per unit per annum) (10)
- OR
- (p) (i) Obtain an expression for present value of deferred immediate annuity certain and in usual notations show that $a_{m+n|} - a_{m|} = m a_{n|}$ where m is deferment period. (Assume rate of interest is i per unit per annum) (07)
- (ii) In usual notations show that $-\frac{a_{6n|}}{a_{3n|}} = 1 + v^{3n}$ (03)

- (q) Define the following terms- (10)
(i) perpetuity
(ii) Immediate perpetuity
(iii) perpetuity due

Hence obtain expression for present value of deferred perpetuity due and deferred immediate perpetuity of Rs. 1 p.a. with deferment period of m years. (Assume rate of interest is i per unit per annum)

- Q.3. (a) Define Deferred immediate life annuity. Obtain the expression for the present value of deferred immediate life annuity of Rs. 1 p.a. to a person aged x in terms of commutation functions. (10)

- (b) Define life annuity due. Obtain the expression for present value of life annuity due of Rs. 1 p.a. on a life now aged x for ' n ' years in terms of commutation functions. (10)

OR

- (p) Define the term temporary life annuity. How is it different from annuity certain? Obtain the expression for the present value of immediate temporary life annuity of Rs. 1 p.a. to a person aged x years in terms of commutation functions. (10)

- (q) Obtain the expression for the present value of increasing life annuity when the payments are made at the end of each year to a person aged x . Write the result in terms of commutation functions. (10)

- Q.4. (a) Obtain the expression for the level annual premium for temporary assurance in terms of commutation functions after deriving all the necessary results. (10)

- (b) Compute the present value of Pure endowment assurance plan. Derive it's the level annual premium in terms of commutation functions. (10)

OR

- (p) Obtain the present value of special endowment assurance plan. Derive the expression for level annual premium in terms of commutation functions for this plan. (10)

- (q) Derive the expression for the level annual premium for whole life assurance in terms of commutation functions after deriving all the necessary results. (10)

Q.5.

Attempt ANY TWO sub-questions.

- (a) (i) Prove that- Probability that among three live all aged x years, the first death will occur in $(t+1)^{\text{th}}$ year is given by $({}_tP_x)^3 - ({}_{t+1}P_x)^3$ (06)
- (ii) State uses of life tables. (04)
- (b) Obtain an expression for accumulated value of immediate annuity of Rs. 1 p.a. payable p times a year for n years certain. (Assume rate of interest is i per unit per annum) (10)
- ~~(c)~~ Derive the expression for the present value of deferred life annuity due of Rs. 1 p.a. payable to a person aged x in terms of commutation functions. (10)
- ~~(d)~~ Explain the concept of limited payment assurance with the help of endowment assurance plan. Derive the level annual premium for the same in terms of commutation functions. (10)
-